1. The objective of this problem is to prove that, with respect to the Theorem of Graham & Brent, a greedy scheduler achieves the stronger bound:   
     
   Let be the representing the instruction stream for a multithreaded program in the fork-join parallelism model. The sets and denote the vertices and edges of respectively. Let and be the work and span of the corresponding multithreaded program. We assume that is connected. We also assume that admits a single source (vertex with no predecessors) denoted by and a single target (vertex with no successors) denoted by . Recall that is the total number of elements of and is the maximum number of nodes on a path from to (counting and ).  
     
   Let . For , we denote by the set of the vertices satisfying the following two properties:  
   1. all immediate predecessors of belong to
   2. at least one immediate predecessor of belongs to .

Therefore, the set represents all the unites of work which can be done during the -th parallel step (and not before that point) on infinitely many processors.  
  
Let be an integer. For all , we denote by the number of elements in . Let be the largest integer such that . Observe that from a partition of . Finally, we define the following sequence of integers:

1. For the computation of the 5-th Fibonacci number (as studied in class) what are ?

../../../Downloads/4402-Problem1-question1.png

1. Show that and both hold.

is the number of nodes on the largest path between . Furthermore, it is observed that form a partition of by level. It is also observed that for any level there exists at least one immediate predecessor of where that belongs to meaning that the longest path between must contain at least one vertex of in every level. Therefore,  
  
Additionally, is the largest integer such that . Consequently, is the where . Hence,   
  
\*\*The is added as an accomidation for the counter starting at .

is the number of vertices in the DAG, . Furthermore, it is observed that form a partition of , therefore,  
And,  
Additionally, since , where , represents all the unites of work which can be done during the -th parallel step on infinitely many processors. Then,  
  
Hence,

1. Show that we have:

Given,

Upon investigating the equation of closely, it becomes clear that calculates the number of cycles required to complete a step upon encountering an incomplete step in a greedy scheduler. Therefore,

Hence,

1. Prove the desired inequality:

With the help of the inequality deduced in the question 3.

The inequality could be easily proven via rearrangement

1. Application; Professor Brown takes some measurements of his (deterministic) multithreaded program, which is scheduled using a greedy scheduler and finds that seconds and seconds. Give lower bound and an upper bound for Professor Brown’s computation running time on processors, for ? Using a plot is recommended.

Using the Theorem of Graham & Brent,

The value of , , allows us to transform the inequality to an equation because is small. Therefore, the following system of equations can be used to determine the values of and ,

Hence,

Finally, to determine the upper and lower bounds of the program plot 100 and 1 for the lower bound and upper bound respectively